5-2 Direct Variation

Content Standards

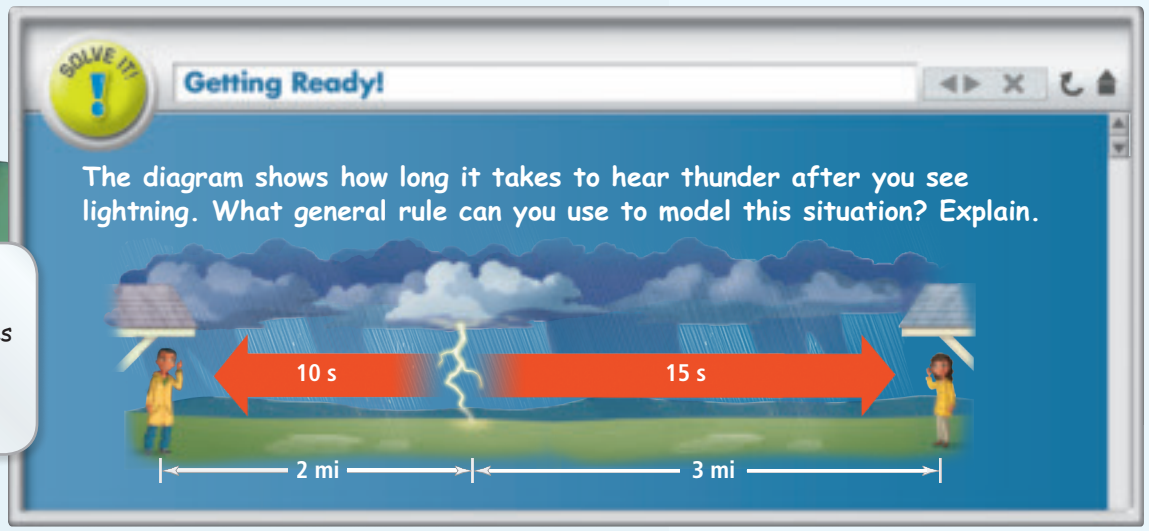
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Also N.Q.2

Objective To write and graph an equation of a direct variation

As your distance from lightning increases, so does the time it takes you to hear the thunder.

MATHEMATICAL PRACTICES



The time it takes to hear thunder *varies directly with* the distance from lightning.

Dynamic Activity
Direct Variation

Essential Understanding If the ratio of two variables is constant, then the variables have a special relationship, known as a *direct variation*.

Lesson Vocabulary
- direct variation
- constant of variation for a direct variation

A **direct variation** is a relationship that can be represented by a function in the form $y = kx$, where $k \neq 0$. The **constant of variation for a direct variation** k is the coefficient of x. By dividing each side of $y = kx$ by x, you can see that the ratio of the variables is constant: $\frac{y}{x} = k$.

To determine whether an equation represents a direct variation, solve it for y. If you can write the equation in the form $y = kx$, where $k \neq 0$, it represents a direct variation.

Problem 1 Identifying a Direct Variation

Does the equation represent a direct variation? If so, find the constant of variation.

Think
Do these equations look like ones you've seen before?
Yes. They contain two variables, so they're literal equations. To determine whether they're direct variation equations, solve for y.

A $7y = 2x$

$y = \frac{2}{7}x$ ← Solve each equation for y. →

The equation has the form $y = kx$, so the equation is a direct variation. Its constant of variation is $\frac{2}{7}$.

B $3y + 4x = 8$

$3y = 8 - 4x$

$y = \frac{8}{3} - \frac{4}{3}x$

You cannot write the equation in the form $y = kx$. It is not a direct variation.

Got It? **1.** Does $4x + 5y = 0$ represent a direct variation? If so, find the constant of variation.

To write an equation for a direct variation, first find the constant of variation k using an ordered pair, other than $(0, 0)$, that you know is a solution of the equation.

Problem 2 Writing a Direct Variation Equation

Suppose y varies directly with x, and $y = 35$ when $x = 5$. What direct variation equation relates x and y? What is the value of y when $x = 9$?

Think

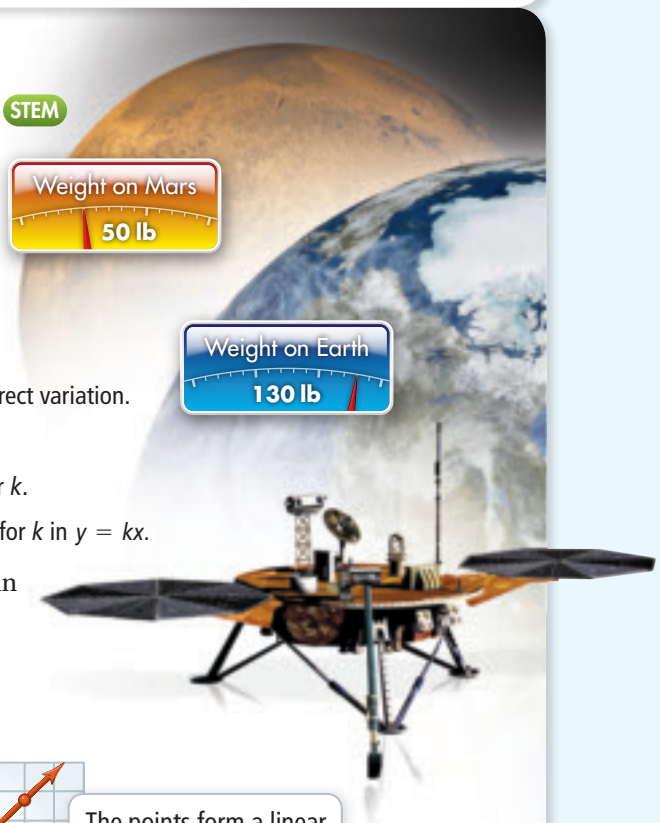
Make sure you don't stop at $7 = k$. To write the direct variation equation, you have to substitute 7 for k in $y = kx$.

$y = kx$	Start with the function form of a direct variation.
$35 = k(5)$	Substitute 5 for x and 35 for y.
$7 = k$	Divide each side by 5 to solve for k.
$y = 7x$	Write an equation. Substitute 7 for k in $y = kx$.

The equation $y = 7x$ relates x and y. When $x = 9$, $y = 7(9)$, or 63.

Got It? **2.** Suppose y varies directly with x, and $y = 10$ when $x = -2$. What direct variation equation relates x and y? What is the value of y when $x = -15$?

Problem 3 Graphing a Direct Variation STEM



Space Exploration **Weight on Mars y varies directly with weight on Earth x. The weights of the science instruments onboard the Phoenix Mars Lander on Earth and Mars are shown.**

A **What is an equation that relates weight, in pounds, on Earth x and weight on Mars y?**

$y = kx$	Start with the function form of a direct variation.
$50 = k(130)$	Substitute 130 for x and 50 for y.
$0.38 \approx k$	Divide each side by 130 to solve for k.
$y = 0.38x$	Write an equation. Substitute 0.38 for k in $y = kx$.

Think

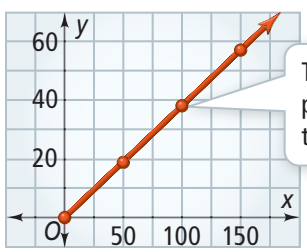
Have you graphed equations like $y = 0.38x$ before?
Yes. In Chapter 4, you graphed linear functions by making a table of values and plotting points.

The equation $y = 0.38x$ gives the weight y on Mars, in pounds, of an object that weighs x pounds on Earth.

B **What is the graph of the equation in part (A)?**

Make a table of values. Then draw the graph.

x	y
0	$0.38(0) = 0$
50	$0.38(50) = 19$
100	$0.38(100) = 38$
150	$0.38(150) = 57$



The points form a linear pattern. Draw a line through them.

Got It? **3. a.** Weight on the moon y varies directly with weight on Earth x. A person who weighs 100 lb on Earth weighs 16.6 lb on the moon. What is an equation that relates weight on Earth x and weight on the moon y? What is the graph of this equation?

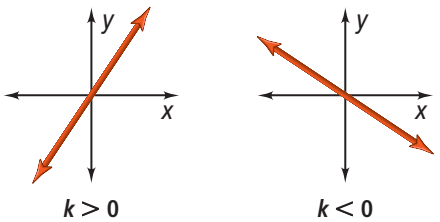
b. Reasoning What is the slope of the graph of $y = 0.38x$ in Problem 3? How is the slope related to the equation?

take note

Concept Summary Graphs of Direct Variations

The graph of a direct variation equation $y = kx$ is a line with the following properties.

- The line passes through (0, 0).
- The slope of the line is k.



You can rewrite a direct variation equation $y = kx$ as $\frac{y}{x} = k$. When a set of data pairs (x, y) vary directly, $\frac{y}{x}$ is the constant of variation. It is the same for each data pair.

Problem 4 Writing a Direct Variation From a Table

For the data in the table, does y vary directly with x? If it does, write an equation for the direct variation.

A

x	y
4	6
8	12
10	15

Find $\frac{y}{x}$ for each ordered pair.

$\frac{6}{4} = 1.5 \quad \frac{12}{8} = 1.5 \quad \frac{15}{10} = 1.5$

The ratio $\frac{y}{x} = 1.5$ for each data pair. So y varies directly with x. The direct variation equation is $y = 1.5x$.

B

x	y
−2	3.2
1	2.4
4	1.6

Find $\frac{y}{x}$ for each ordered pair.

$\frac{3.2}{-2} = -1.6 \quad \frac{2.4}{1} = 2.4 \quad \frac{1.6}{4} = 0.4$

The ratio $\frac{y}{x}$ is not the same for all data pairs. So y does not vary directly with x.

Plan

How can you check your answer?
Graph the ordered pairs in the coordinate plane. If you can connect them with a line that passes through (0, 0), then y varies directly with x.

Got It? **4.** For the data in the table at the right, does y vary directly with x? If it does, write an equation for the direct variation.

x	y
−3	2.25
1	−0.75
4	−3

Lesson Check

Do you know HOW?

1. Does the equation $6y = 18x$ represent a direct variation? If it does, what is its constant of variation?
2. Suppose y varies directly with x, and $y = 30$ when $x = 3$. What direct variation equation relates x and y?
3. A recipe for 12 corn muffins calls for 1 cup of flour. The number of muffins you can make varies directly with the amount of flour you use. You have $2\frac{1}{2}$ cups of flour. How many muffins can you make?
4. Does y vary directly with x? If it does, what is an equation for the direct variation?

x	y
-2	1
2	-1
4	-2

Do you UNDERSTAND? MATHEMATICAL PRACTICES

Vocabulary Determine whether each statement is *always*, *sometimes*, or *never* true.

5. The ordered pair $(0, 0)$ is a solution of the direct variation equation $y = kx$.
6. You can write a direct variation in the form $y = k + x$, where $k \neq 0$.
7. The constant of variation for a direct variation represented by $y = kx$ is $\frac{y}{x}$.
8. **Reasoning** Suppose q varies directly with p. Does this imply that p varies directly with q? Explain.

Practice and Problem-Solving Exercises

A Practice

Determine whether each equation represents a direct variation. If it does, find the constant of variation. See Problem 1.

9. $2y = 5x + 1$
10. $8x + 9y = 10$
11. $-12x = 6y$
12. $y + 8 = -x$
13. $-4 + 7x + 4 = 3y$
14. $0.7x - 1.4y = 0$

Suppose y varies directly with x. Write a direct variation equation that relates x and y. Then find the value of y when $x = 12$. See Problem 2.

15. $y = -10$ when $x = 2$.
16. $y = 7\frac{1}{2}$ when $x = 3$.
17. $y = 5$ when $x = 2$.
18. $y = 125$ when $x = -5$.
19. $y = 10.4$ when $x = 4$.
20. $y = 9\frac{1}{3}$ when $x = -\frac{1}{2}$.

Graph each direct variation equation. See Problem 3.

21. $y = 2x$
22. $y = \frac{1}{3}x$
23. $y = -x$
24. $y = -\frac{1}{2}x$

25. **Travel Time** The distance d you bike varies directly with the amount of time t you bike. Suppose you bike 13.2 mi in 1.25 h. What is an equation that relates d and t? What is the graph of the equation?
26. **Geometry** The perimeter p of a regular hexagon varies directly with the length ℓ of one side of the hexagon. What is an equation that relates p and ℓ? What is the graph of the equation?

For the data in each table, tell whether y varies directly with x. If it does, write an equation for the direct variation. Check your answer by plotting the points from the table and sketching the line.

See Problem 4.

27.

x	y
−6	9
1	−1.5
8	−12

28.

x	y
3	5.4
7	12.6
12	21.6

29.

x	y
−2	1
3	6
8	11

B Apply

Suppose y varies directly with x. Write a direct variation equation that relates x and y. Then graph the equation.

30. $y = \frac{1}{2}$ when $x = 3$. **31.** $y = -5$ when $x = \frac{1}{4}$. **32.** $y = \frac{6}{5}$ when $x = -\frac{5}{6}$. **33.** $y = 7.2$ when $x = 1.2$.

34. Think About a Plan The amount of blood in a person's body varies directly with body weight. A person who weighs 160 lb has about 4.6 qt of blood. About how many quarts of blood are in the body of a 175-lb person?

- How can you find the constant of variation?
- Can you write an equation that relates quarts of blood to weight?
- How can you use the equation to determine the solution?

STEM **35. Electricity** Ohm's Law $V = I \times R$ relates the voltage, current, and resistance of a circuit. V is the voltage measured in volts. I is the current measured in amperes. R is the resistance measured in ohms.

a. Find the voltage of a circuit with a current of 24 amperes and a resistance of 2 ohms.

b. Find the resistance of a circuit with a current of 24 amperes and a voltage of 18 volts.

Reasoning Tell whether the two quantities vary directly. Explain your reasoning.

36. the number of ounces of cereal and the number of Calories the cereal contains

37. the time it takes to travel a certain distance and the rate at which you travel

38. the perimeter of a square and the side length of the square

39. the amount of money you have left and the number of items you purchase

40. a. Graph the following direct variation equations in the same coordinate plane: $y = x$, $y = 2x$, $y = 3x$, and $y = 4x$.

b. Look for a Pattern Describe how the graphs change as the constant of variation increases.

c. Predict how the graph of $y = \frac{1}{2}x$ would appear.

41. Error Analysis Use the table at the right. A student says that y varies directly with x because as x increases by 1, y also increases by 1. Explain the student's error.

x	y
0	3
1	4
2	5

42. Writing Suppose y varies directly with x. Explain how the value of y changes in each situation.

a. The value of x is doubled.

b. The value of x is halved.

STEM **43. Physics** The force you need to apply to a lever varies directly with the weight you want to lift. Suppose you can lift a 50-lb weight by applying 20 lb of force to a certain lever.

a. What is the ratio of force to weight for the lever?

b. Write an equation relating force and weight. What is the force you need to lift a friend who weighs 130 lb?

Challenge

The ordered pairs in each exercise are for the same direct variation. Find each missing value.

44. $(3, 4)$ and $(9, y)$

45. $(1, y)$ and $\left(\frac{3}{2}, -9\right)$

46. $(-5, 3)$ and $(x, -4.8)$

47. Gas Mileage A car gets 32 mi per gallon. The number of gallons g of gas used varies directly with the number of miles m traveled.

a. Suppose the price of gas is \$3.85 per gallon. Write a function giving the cost c for g gallons of gas. Is this a direct variation? Explain your reasoning.

b. Write a direct variation equation relating the cost of gas to the miles traveled.

c. How much will it cost to buy gas for a 240-mi trip?

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

48. The price p you pay varies directly with the number of pencils you buy. Suppose you buy 3 pencils for \$.51. How much is each pencil, in dollars?

49. A scooter can travel 72 mi per gallon of gasoline and holds 2.3 gal. The function $d(x) = 72x$ represents the distance $d(x)$, in miles, that the scooter can travel with x gallons of gasoline. How many miles can the scooter go with a full tank of gas?

50. The table at the right shows the number of hours a clerk works per week and the amount of money she earns before taxes. If she worked 34 h per week, how much money would she earn, in dollars?

Weekly Wages

Time (h)	Wages (\$)
12	99.00
17	140.25
21	173.25
32	264.00

51. What is the greatest value in the range of $y = x^2 - 3$ for the domain $\{-3, 0, 1\}$?

Mixed Review

Find the slope of the line that passes through each pair of points. See Lesson 5-1.

52. $(2, 4), (0, 2)$

53. $(5, 8), (-5, 8)$

54. $(0, 0), (3, 18)$

55. $(1, -2), (-2, 3)$

Get Ready! **To prepare for Lesson 5-3, do Exercises 56–59.**

Evaluate each expression. See Lesson 1-2.

56. $6a + 3$ for $a = 2$

57. $-2x - 5$ for $x = 3$

58. $\frac{1}{4}x + 2$ for $x = 16$

59. $8 - 5n$ for $n = 3$